

# Physics informed machine learning for turbulence modeling

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## 1 Background

Turbulence is among the last unsolved problems in classical physics, and it impacts many issues of societal importance including energy, environment, and climate. Accurate predictions of turbulent flows are of vital importance for the design and operation of mission-critical systems such as aircraft, gas turbine engines, and nuclear power plants<sup>[1-3]</sup>. Currently, RANS simulations are still the workhorse simulation tool for industrial turbulent flows, as direct numerical simulations and large eddy simulations (LES) are still too expensive computationally<sup>[4]</sup>. RANS simulations rely on turbulence models to represent the unresolved physics. These models introduce large uncertainties into the results, severely impairing their predictive capabilities<sup>[5-9]</sup>. Such difficulties have been highlighted in recent reviews<sup>[10-11]</sup>.

Development of turbulence models has been stagnant for decades, which is evident from two observations. First, the number of costly wind tunnel tests performed in a typical design cycle of a commercial airplane was reduced from 75 in the 1970s to 10 in the 1990s, but this number has been stagnant since then, with turbulence models being the major bottleneck in predictive accuracies<sup>[5]</sup>. Second, currently used turbulence models ( $k-\varepsilon$ <sup>[12]</sup>,  $k-\omega$ <sup>[13]</sup>, Spalart-Allmaras<sup>[14]</sup>) were all developed decades ago despite unsatisfactory performance for many flows. Generations of researchers have labored for many decades on dozens of turbulence models, yet none of them achieved predictive generality. Flow-specific tuning and “fudge functions” are still an indispensable part of RANS simulations<sup>[15]</sup>.

Recently, researchers have attempted using machine learning to augment turbulence models. For example, Duraisamy et al.<sup>[16-18]</sup> introduced a multiplicative discrepancy field to the source term of the turbulence transport equations. Ling et al.<sup>[19]</sup> proposed a tensor basis neural network based on Pope’s general algebraic stress model and learned the coefficients therein from DNS databases. Zhang et al.<sup>[20]</sup> utilized such a methodology to investigate the plane channel flows achieve successful predictions. Weatheritt and Sandsberg<sup>[21-22]</sup> used symbolic regression and gene expression programming for learning the coefficients in algebraic turbulence models. In this work, we introduce and demonstrate the procedures toward a complete machine learning framework for predictive turbulence modeling, including learning Reynolds stress discrepancy function, predicting Reynolds stresses in different flows, and propagating the predicted Reynolds stresses to mean flow fields.

## 2 Methodology

The aim of the present work is to introduce and demonstrate the physics-informed machine learning (PIML) framework for predictive turbulence modeling. Specifically, given high-fidelity data (e.g., Reynolds stresses from DNS simulations) from a set of training flows, the framework aims to improve the standard RANS prediction for different flows for which DNS data are not available. As illustrated in Fig. 1, there are four essential components in the PIML framework: (1) construction of the input feature set, (2) representation of the Reynolds stress discrepancy as the response, (3) construction of the regression function of the discrepancy with respect to input features, and (4) propagation of corrected Reynolds stresses to mean velocities. As in traditional constitutive modeling, a data-driven constitutive model should have invariance under Galilean transformation and coordinate rotation. The frame-independence requirement is satisfied by properly choosing inputs (mean flow features  $\mathbf{q}$ ) and outputs (discrepancies of the modeled Reynolds stresses) for the machine learning<sup>[23]</sup>. Details of the method are presented in our previous works<sup>[23-24]</sup>.

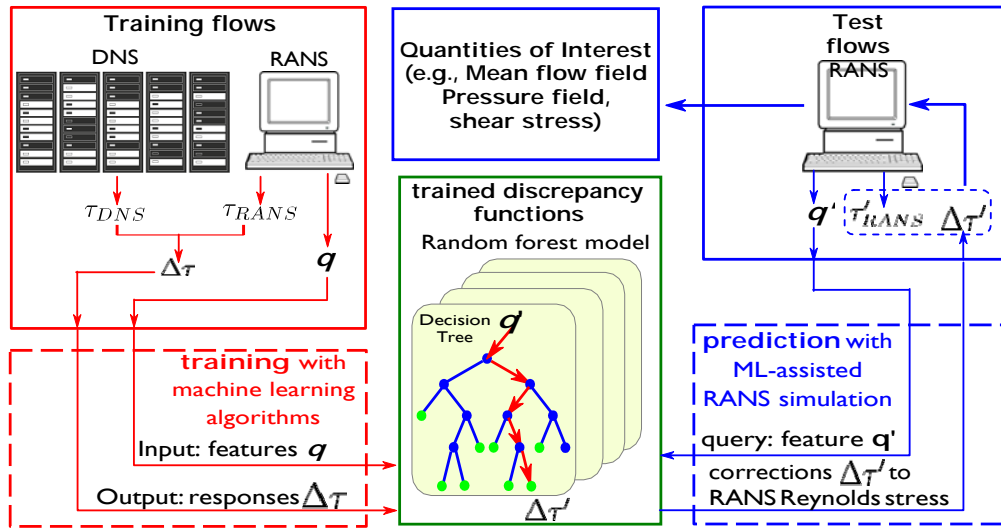


Figure 1: Schematic of Physics-Informed Machine Learning (PIML) framework for predictive turbulence modeling

## 3 Numerical Results

We have tested our method on a number of simple flows, including square duct flows, periodic hill flows<sup>[23-24]</sup>, and high Mach number flat boundary layer flows<sup>[25]</sup>. Here we present the results of flow over periodic hills as an example. The test flow is the flow over periodic hills at  $Re = 5600$ , and the training flow is the flow with a steeper hill profile. The comparison of mean velocity

field in Fig. 2 shows that the mean velocity obtained by the machine-learning-assisted turbulence modeling framework has a better agreement with the DNS data, particularly in the recirculation region. Compared with the RANS simulation results, our machine-learning-assisted turbulence model predicts a flow pattern that agrees much better with the DNS data.

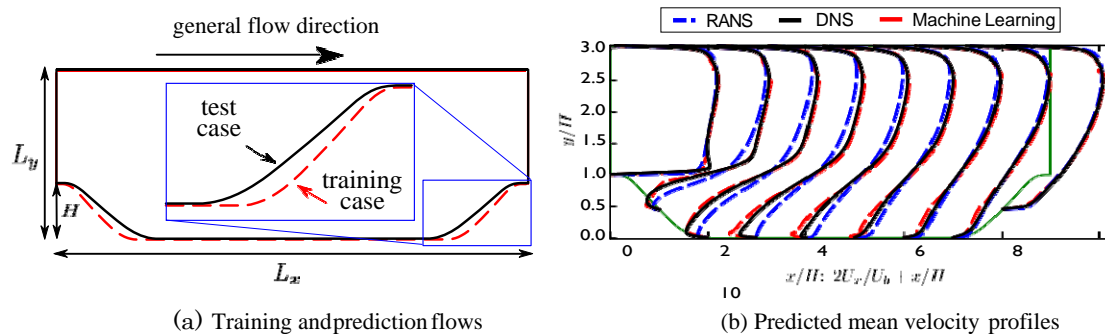


Figure 2 The preliminary results of the flow over periodic hills. (a): computational domain with a zoom-in view of hill profiles for both training and test flows. (b): predicted mean flow velocity

## 4 Conclusion

In view of the decades long stagnation in turbulence modeling, we present a comprehensive framework for augmenting turbulence models with physics-informed machine learning, illustrating a complete workflow from identification of input features to final prediction of mean velocities. The proposed method has been tested on a number of canonical flows and has achieved preliminary successes.

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